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Marangoni effects in welding

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The problem of variable weld penetration or 'cast-to-cast' variation in GTA/TIG welding is discussed. It is shown that for normal GTA/TIG welding conditions the Heiple–Roper theory is valid, i.e. that weld penetration is controlled by the fluid flow in the weld pool which, in turn, is controlled by the direction and magnitude of the thermocapillary forces. For most steels the direction and magnitude of these forces are determined by the sulphur content, since the temperature coefficient of surface tension $(d\gamma/dT)$ is negative when S < 30 ppm and this leads to a radially outward flow and poor penetration whereas a steel with S > 60 ppm has a positive $(d\gamma/dT)$ which produces a radially inward flow giving good weld penetration. Thermocapillary forces were shown to play a part in the problems of 'off-centre welding', 'porosity' and 'arc wander' in GTA/TIG welding and in the surface rippling of welds.

Keywords: TIG/GTA welding; penetration; thermocapillary forces; sulphur content

1. Introduction

The first example where Marangoni forces have been proposed to be involved in welding problems was in the case of 'cast-to-cast' variations or variable weld penetration.

The problem of 'cast-to-cast' variations in weld penetration produced during autogenous tungsten inert gas (TIG)[†] welding of stainless and ferritic steels was first noted in the 1960s. The problem is particularly severe in robotic processes requiring thousands of repetitive welds where it is customary to establish the welding parameters which promote deep penetration joints. However, it has been found (as can be seen in figure 1) that certain batches of steel produced welds with much lower weld penetration than the norm, despite fully meeting the material specifications. Welds can be partial-penetration welds (figures 1a, b) or full-penetration welds (figure 1c). It is customary to express penetration in partial welds by the depth (D)/width (W)ratio.

There have been several attempts to establish a correlation between 'cast-to-cast' variations and systematic variations in the concentrations of specific minor or impurity elements in the metal. However, where such a relationship could be identified it was noted that any such variations in the element concentrations were very small. Thus, any theory proposed to account for variable weld penetration must explain why such small differences in chemical composition can have such a large effect on 'weldability'. Several theories have been proposed in which it was suggested that small differences in minor element concentrations in the steel produced changes in

[†] Sometimes known as gas tungsten arc (GTA) welding in which an arc is struck between a cathode and the workpiece (anode).

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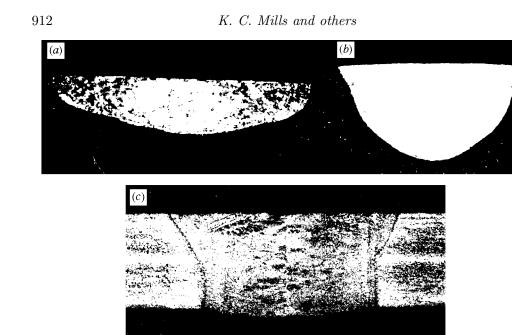


Figure 1. Comparison of the cross section of TIG weld fusion zones in (a) shallow- and (b) deep-penetration welds in stainless steel (×10) and (c) full-penetration weld, this case in penetration defined by the ratio of widths of back and front welds $(W_{\rm b}/W_{\rm f})$.

(a) the arc characteristics (Glickstein *et al.* 1977; Savage *et al.* 1977) and (b) the surface properties of the weld pool by affecting either the interfacial energies (Roper & Olsen 1978) or the fluid flow motion in the weld pool (Heiple & Roper 1982). However, it has also been found that variable weld penetration occurred in non-arc processes such as laser and electron beam welding (Robinson *et al.* 1982; Heiple *et al.* 1983) where there are no arc effects. Consequently, cast-to-cast variation could not be explained solely on the basis of changes in the arc characteristics and thus attention has been focused mostly on changes in the surface properties of the melt.

As can be seen from figure 2, small differences in the concentrations of surfaceactive elements, such as sulphur (Gupt *et al.* 1972/73) and oxygen (Gupt *et al.* 1976), cause substantial changes in the surface tension (γ) of iron and other elements. Friedman (1978) developed a model of the weld pool in which it was proposed that the surface-tension forces operating in the pool opposed the combined effects of gravity and arc pressure; thus a high surface tension would lead to poor weld penetration. Other theories have focused on the effect of the surface tension on the fluid flow in the weld pool. Ishizaki (1965) suggested that the surface-tension gradient ($d\gamma/dT$) across the pool could affect the convective flow in the weld pool. Heiple & Roper (1982) developed this theory, and postulated that variable weld penetration is a result of differences in the fluid flow in the weld pool resulting from differences in both the direction and magnitude of thermocapillary forces, and that these were controlled by the concentrations of surface-active elements such as sulphur and oxygen in the metal.

Heiple & Roper (1982) also pointed out that when the sulphur or oxygen concentration exceeded a certain critical value (around 50 ppm), the temperature coefficient of surface tension $(d\gamma/dT)$ changed from a negative to a positive value (figure 3). They suggested that since a large temperature gradient exists between the centre and

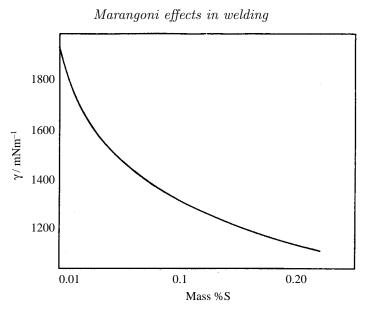


Figure 2. Variation of surface tension with sulphur content (Gupt et al. 1972/73).

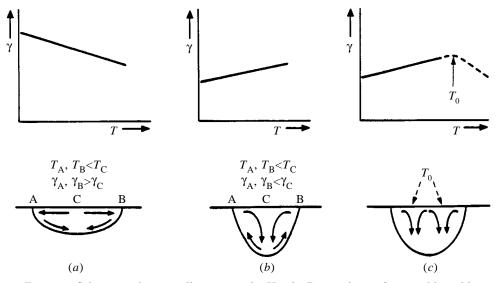


Figure 3. Schematic diagram illustrating the Heiple–Roper theory for variable weld penetration (Heiple & Roper 1982).

the edges of the weld pool (of the order of 500 K mm⁻¹), a large surface-tension (γ) gradient will be produced across the surface. The resulting Marangoni flow will occur from a region of low γ to a region of high γ . These surface flows subsequently trigger circulation flows in the molten weld pool, as shown in figure 3. For most pure metals, including iron and steels with low O and S contents, the surface tension decreases with increasing temperature, which results in a negative surface-tension-temperature coefficient $(d\gamma/dT)$ (figure 3a). In this case, the surface tension will be greatest in the cooler regions at the edge of the weld pool and this induces a radially outward surface flow which carries hot metal to the edge of the pool where the consequent melt-back results in a wide shallow weld. In contrast to this, in Fe-based melts with

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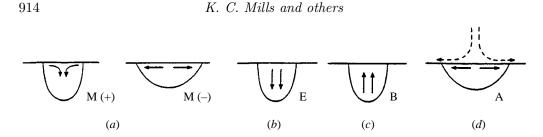


Figure 4. (a) Thermocapillary (Marangoni) forces M(+) or M(-); (b) electromagnetic (Lorentz) forces E, resulting from interaction of current; (c) buoyancy forces B, resulting from density differences caused by temperature gradients; (d) aerodynamic drag forces A, caused by passage of plasma over surface.

S (or O) > 60 ppm, $(d\gamma/dT)$ will be positive (figure 3b) and thus the surface tension is greatest in the high-temperature region at the centre of the pool and this induces a radially inward flow. This, in turn, produces a downward flow in the centre of the weld pool (figure 3b) which transfers hot metal to the bottom of the pool where melt-back of the metal results in a deep and narrow pool.

Keene *et al.* (1982) have pointed out that systems which exhibit a positive $(d\gamma/dT)$ must go through a maximum at some temperature and thus produce a complex flow similar to that shown in figure 3c.

2. Forces affecting the fluid flow in the weld pool

The Heiple–Roper theory makes two assumptions: (i) that the heat transfer in the weld pool is controlled by the fluid flow in the pool and not the heat conduction in the workpiece; and (ii) that the fluid flow is dominated by the thermocapillary forces.

However, there are several other fluid flow mechanisms operating in the weld pool, namely electromagnetic (or Lorentz), aerodynamic drag and buoyancy forces (figure 4). Under certain welding conditions these forces can have a significant effect on the fluid flow in the weld pool.

(a) Marangoni forces

These are, for the most part, thermocapillary forces but diffusocapillary forces can arise when welding steels with different sulphur contents (see $\S7a$). The direction of the thermocapillary flow is determined by the concentration of O or S in the alloy.

The strength of the thermocapillary flow is determined by the non-dimensional Marangoni number (Ma) defined in equation (2.1) where (dT/dx) is the temperature gradient, η is the viscosity, a is the thermal diffusivity and L is the characteristic length:

$$Ma = \frac{\mathrm{d}\gamma}{\mathrm{d}T} \frac{\mathrm{d}T}{\mathrm{d}x} \frac{L^2}{\eta a}.$$
(2.1)

(b) Electromagnetic or Lorentz forces

The Lorentz forces are caused by the interaction of the induced magnetic field and the current carried by a conductor. The welding current induces a magnetic field around the conductor and the Lorentz force acts inwards and downwards in the weld pool (figure 4b).

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(c) Buoyancy forces

Buoyancy forces are caused by the density differences due to temperature gradients in the weld pool and result in an upward flow (figure 4c). However, it has been shown that buoyancy forces are generally very small in relation to the other forces in weld pools of less than 10 mm depth.

(d) Aerodynamic drag forces

These forces are produced by the action of the arc plasma flowing over the surface of the weld pool, which induce an outward flow along the surface of the pool (figure 4d).

However, the fluid flow in the weld pool is exceedingly complex since, thermocapillary, Lorentz, aerodynamic and buoyancy forces can all influence the flow. The situation is further complicated by (i) the front-to-back flow resulting from the relative motion of the workpiece to that of the electrode which is particularly important at high welding speeds and (ii) the 'spin' developed by the liquid metal under conditions of radially inward flow, which tends to reduce the magnitude of the radially inward flow (Lancaster 1987).

The mathematical modelling of the relative strengths of the four forces affecting the fluid flow has become a subject of great interest in recent years and about twenty models have been reported (Mills & Keene 1990). Virtually all these models predict that the Marangoni forces are predominant under normal welding conditions and have a decisive effect on the weld profile (Oreper & Szekely 1984; Kou & Sun 1985).

3. Relation between penetration and surface tension

As mentioned previously, penetration is usually expressed by the ratios of the (depth/width) (D/W) and the (back/front) widths (W_b/W_f) for partial- and full-penetration welds, respectively; the latter parameter is subject to welding characteristics and is a less satisfactory measure than (D/W).

The link between weld penetration and surface tension of the alloy has been demonstrated by Mills *et al.* (1984) who used the levitated drop method to measure the surface tension of casts with good and bad penetration, i.e. high and low (D/W)ratios, respectively. A typical example is shown in figure 5*a*. It was found that:

(i) good weld penetration correlated with low values of surface tension (γ) and positive values of $(d\gamma/dT)$ as found in steels with high-sulphur (HS) contents;

(ii) poor weld penetration correlated with high values of γ and negative values of $(d\gamma/dT)$ as found in steels with LS contents.

Mills & Keene (1990) subsequently correlated $(d\gamma/dT)$ with the S contents of the steels and showed that the 'cross-over' point where $(d\gamma/dT) = 0$ occurred around 40 ppm S (figure 5b). Thus good weld penetration is obtained in steels with greater than 60 ppm S and poor penetration in casts with less than 30 ppm S.

Mathematical models have shown that fluid flow in the weld pool is complex, despite the fact that thermocapillary forces tend to be dominant. Nevertheless, on the basis of the Heiple–Roper theory, some correlation between (D/W) and $(d\gamma/dT)$ might be expected. The γ –T relationship was determined for three steels with different S contents and the (D/W) ratio was derived for partial welds carried out on 6 mm plates of these steels. It can be seen from figure 6 that there is a good correlation between (D/W) and $(d\gamma/dT)$.

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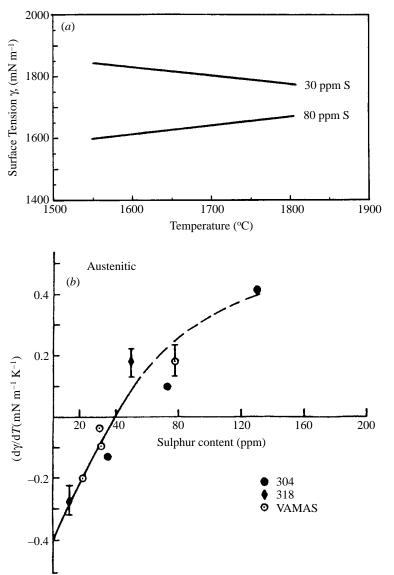


Figure 5. (a) Surface tension of stainless steels with low sulphur (30 ppm) giving poor penetration and high sulphur (80 ppm) and good penetration and (b) dependence of $d\gamma/dT$ on sulphur content.

4. Effect of various elements on weld penetration

Elements can be classified into the following three classes.

(i) *Surface-active* elements (eg S, O, Se, Te), which affect the magnitude and direction of the fluid flow.

(ii) *Reactive* elements (eg Ca, Ce, Al), which react with the surface-active elements and thereby reduce the concentrations of soluble O and S.

(iii) Neutral elements which have little effect on the fluid flow in the weld pool.

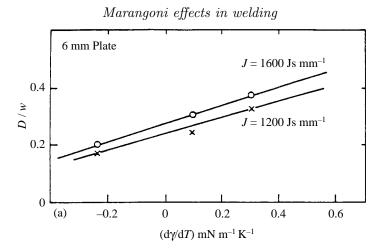


Figure 6. Penetration (D/W) as a function of $(d\gamma/dT)$ for 6 mm-thick plates.

(a) Surface-active elements

(i) Oxygen

Although oxygen is almost as surface active as sulphur, Robinson & Gould (1987) have shown that it does not always have as great an effect on weld penetration as sulphur. It should be noted that it is the concentration of *soluble* O or S, denoted \underline{O} , or \underline{S} , which affects the surface tension since the *combined* oxygen (in the form of oxides) has little effect on the surface tension. However, it can be seen from figures 7*a*, *b* that the concentrations of scavenging elements, such as Al, in steel will hold the soluble \underline{O} concentration below 10 ppm, whereas they do not have the same effect on the soluble \underline{S} unless the steel contains large concentrations of Ca or Ce, which is rare. Thus, $\underline{O} \ll O_{\text{total}}$ and $\underline{S} \simeq S_{\text{total}}$ and therefore it is the sulphur and not the oxygen which has the greatest effect on the weld penetration.

(ii) Sulphur

Recent work on the effect of sulphur (Shirali & Mills 1993) on weld penetration was carried out on both high-sulphur (HS) and low-sulphur (LS) steels using various doping techniques; the results are shown in figure 8. It was found that for all the steels, an increase in S content produced an increase in the depth/width (D/W)ratio. These results are in essential agreement with Heiple–Roper theory and with results obtained by other investigators.

(b) Reactive elements

It can be seen from figure 7*a* that Al additions in excess of 20 ppm will react with the soluble oxygen present in the steel to form Al_2O_3 and thus reduce the soluble \underline{O} concentration to a very low level. Under these conditions $(d\gamma/dT)$ would become negative. Consequently, the thermocapillary forces would be expected to produce a radially outward surface flow resulting in a reduction of the (D/W) ratio of the weld. Calcium and cerium behave in a similar manner. Thus, providing there are sufficient amounts of these elements to react with the oxygen present, the soluble \underline{O} concentration in steels will be below 5 ppm.

The Ca, Ce, La will react with soluble \underline{S} to give their respective sulphides and reduce the \underline{S} to very low levels, but this is not the case for Al (which is sited near to the Zr curve in figure 7b). Thus, providing the steel does not contain significant

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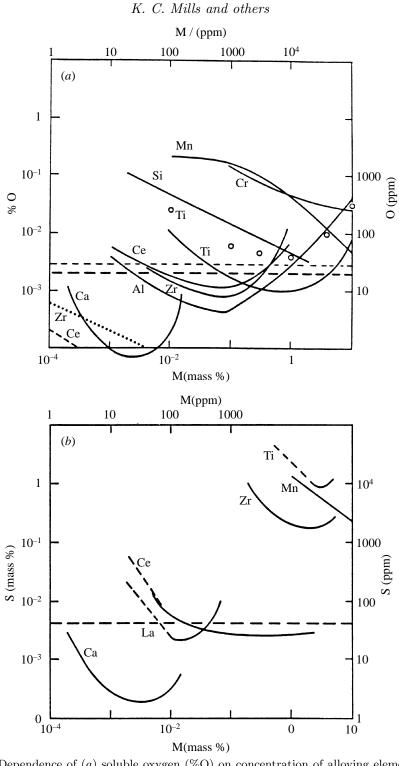


Figure 7. Dependence of (a) soluble oxygen (%O) on concentration of alloying element (M) in Fe–M–O systems at 1873 K; (b) soluble sulphur (%S) on concentration of alloying element (M) in Fe–M–S systems at 1873 K (Mills & Keene 1990).



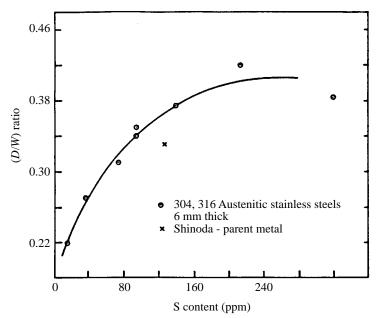


Figure 8. Effect of sulphur content on the (D/W) ratio of TIG welds (Shirali & Mills 1993).

levels of Ca, Ce, La, the soluble \underline{S} concentration will be only slightly less than the total S content. This is the reason why penetration can be readily correlated with S content but is less readily correlated with total O content.

5. Effects of 'slag spots' and oxide films

'Slag spots' are formed by the floating of non-metallic inclusions on the surface of the metal and they are produced by reactions of the metal with O and S. Pollard (1988) showed that they attract the arc and reduce the size of the anode root and thus increase the current density. In casts with S > 50 ppm the fluid flow will be radially inward and the slag will be sited in the centre of the pool; thus the resultant high current density will result in increased temperature gradients and, consequently, better penetration. However, in LS casts the flow will be radially outward and the slag spot will be swept to the periphery of the pool. The consequent attraction of the arc will result in deeper penetration at the edge of the pool, which leads to an erratic weld seam (see §7b).

When the steel contains significant concentrations of Ca (greater than 20 ppm), the Ca forms an oxide film on the edge of the weld pool. These surface films tend to suppress surface flows and thus produce stagnant regions at the edges of the weld pool.

6. Effect of welding parameters on weld penetration

Burgardt & Heiple (1986) pointed out that since Marangoni forces are usually dominant in the weld pool, the effects of altering welding conditions can be explained in terms of what effects these changes would have on the temperature gradient (and hence the strength of Marangoni forces operating in the weld pool). Thus any change

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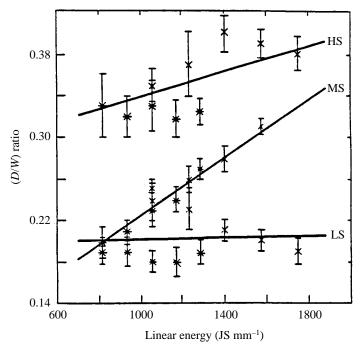


Figure 9. The (D/W) ratio as a function of the linear energy (error bars represent the standard deviation).

which brings about an increase in temperature gradient would cause increased penetration in HS casts and reduced penetration in LS casts. Although this proposition ignores the effect of welding parameters on the other forces operating in the weld pool, these workers did show that it could account for their observations.

Mills and Keene (1990) analysed the effect of changes in the welding parameters, such as arc length, welding speed (S_w) and current (I) and voltage (V), etc., on all four forces affecting fluid flow. The thermocapillary forces are affected by the temperature gradient (dT/dx), which is related to the power input (IV). The travel speed (S_w) affects the rate of heat input to the weld and it is this quantity which controls (dT/dx). Consequently, the effect of welding parameters on the Marangoni forces can best be studied from measurements of the heat input per unit length of weld. This parameter is referred to as the linear energy and is defined as (IV/S_w) .

The linear energy allows the effect of current and welding speed on the Marangoni forces to be taken into account simultaneously but it should be noted that it does not account for either increased Lorentz forces with increasing current or the increased front-to-back motion in the weld pool at high welding speeds. The effect of increased linear energy on high- (HS) medium- (MS) and low-sulphur (LS) steels is shown in figure 9. It can be seen that increases in the linear energy resulted in increased penetration for HS and MS casts but have little effect on the LS casts. Thus these results are in essential agreement with Burgardt & Heiple's (1986) proposition that the effect of welding parameters on penetration can be explained in terms of their effect on the temperature gradients.

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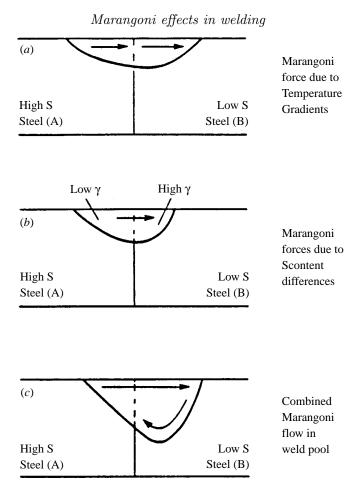


Figure 10. Schematic drawings showing the formation of a non-axisymmetric weld when welding steels with different sulphur contents: (a) Marangoni force due to temperature gradients; (b) Marangoni forces due to S content differences; (c) combined Marangoni flow in weld pool.

7. Other effects in TGI/GTA welding

(a) Off-centre welding

Tinkler *et al.* (1983) showed that when welding a 30 ppm S plate to a 90 ppm S plate, the resulting weld was off-centre and displaced towards the LS side. This can be accounted for if it is assumed that Marangoni forces dominate the fluid flow in the weld pool. It can be seen from figure 10 that the thermocapillary forces in the LS and HS will be from left to right and the diffusocapillary forces will also operate from left to right. Thus these surface flows will cause hot metal to be carried to the LS side and melt back off the steel will result in an asymmetric weld.

(b) Arc wander

It has been mentioned above that certain casts of steel exhibiting poor weld penetration tend to give an erratic weld seam. This is known as 'arc wander' and can be seen in figure 11. It frequently occurs with steels containing greater than 20 ppm Ca. It is caused by slag spots attracting the arc. For radially inward flows the slag spot will be centred in the centre of the pool and the attraction of the arc will result, sequentially, in a greater current density (because of the smaller anode root) and

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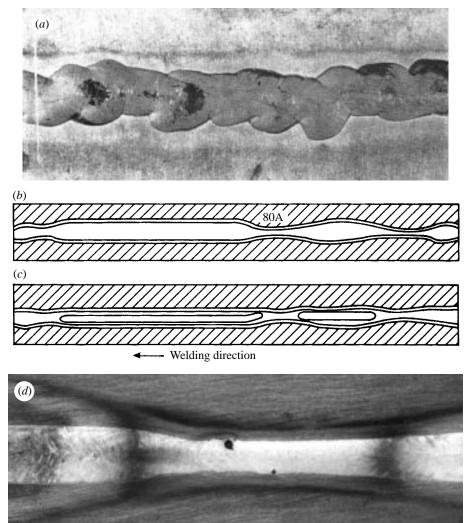


Figure 11. Examples of arc wander: (b) and (c) show the front and back faces of a full penetration weld; (d) is a magnified section of (b) showing presence of slag spots.

deeper penetration. With a radially outward flow the slag spot will be swept to the edge of the pool, since it attracts the arc, the hottest part of the pool will become the region close to slag spot and thus the position of the weld seam will change.

(c) Porosity

Poor weld penetration is often accompanied by porosity. Kou & Wang (1986) proposed that the direction of the flow in the weld pool could be responsible for the presence of pores in the weld. When the flow is radially inward the weld pool motion will assist the escape of bubbles away from the solidification front (figure 12a). In contrast, when the flow is radially outward the bubbles will tend to be swept towards the solidification front (figure 12b). It is obvious that the presence of a solid–slag film at the rear of the pool will tend to create a stagnant region which will assist the

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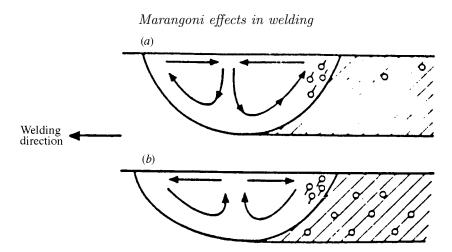


Figure 12. Influence of weld pool motion on porosity: (a) inward surface flow; (b) outward surface flow.



Figure 13. Predicted surface profiles of weld showing humping and undercutting: (a) inward surface flows with downward flow in centre; (b) outward surface flows with downward flow at periphery of weld pool.

entrapment of gas bubbles by the solidification front, thus it is not surprising that porosity problems are encountered when welding steels with high Ca levels.

(d) 'Humping' and 'undercutting'

'Humping' is the formation of a raised section in the centre of the weld and 'undercutting' is the depression at the edge of the weld (figure 13). 'Humping' and 'undercutting' were found to be prevalent for HS casts and when using high travel speeds. One characteristic of Marangoni flow is that the surface is raised in regions where the liquid is being driven downwards and depressed where the flow is upwards (figure 13). Thus for HS casts with a radially inward flow, it is obvious Marangoni flows can account for both humping and undercutting. However, on the basis of the flow patterns shown in figure 13 it is difficult to account for the undercutting in LS casts.

Gratzke *et al.* (1992) rejected the thermocapillary mechanism and proposed that humping and undercutting were caused by Rayleigh instability, i.e. the break-up of a liquid cylinder by the action of surface and gravity forces. They concluded that (i) the (width/length) ratio of the pool was the most important factor and (ii) the surface tension does not affect the onset of humping, only the kinetic behaviour which is a function of $(\rho/\gamma)^{1/2}$. The latter conclusion would seem to be inconsistent with the observation that it was prevalent in HS casts.

(e) Surface rippling

The surfaces of weld pools produced by LS casts tend to be flat and placid in contrast to the surfaces of HS casts which tend to be turbulent and agitated. The solidified welds of both HS and LS alloys exhibit regularly spaced fine ripples, possibly caused by the oscillation frequency of the arc or laser. However, for the HS casts

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there is a series of deeper ripples of longer wavelength superimposed on the fine ripple background. In full penetration welds the coarse ripples were seen on both free surfaces and pulsation of the laser would not be expected to affect the back surface. If the rippling is associated with the surface properties of the steel melt, then it must be related to a low surface tension and a positive $(d\gamma/dT)$. In the weld pool there are also thermal gradients in the direction perpendicular to the surface. Although these gradients do not produce any substantial fluid flow in the weld pool they can produce thermocapillary instabilities. These instabilities arise when a metal with a negative $(d\gamma/dT)$ is heated from below or a metal with positive $(d\gamma/dT)$ is heated from above and which give rise to capillary waves (Nemchinsky 1997). The thermocapillary forces acting parallel to the surface amplify these capillary waves and produce instabilities. It also explains why they only occur in HS casts since they have positive $(d\gamma/dT)$ coefficients and welding is usually carried out by heating the upper surface.

8. Gas–metal arc welding (GMA)

This process is similar to GTA welding but a filler metal is used. Takasu & Toguri (this volume) have shown that there are four forces affecting the fluid motion in the pool when the molten filler metal drop hits the molten pool, namely: (i) a stirring force due to the momentum of the drop; (ii) a buoyancy force related to the density difference between the drop and the pool; (iii) a 'curvature' force related to the surface tension normal to the surface; and (iv) the Marangoni force related to the difference in surface tension of the drop and pool. Takasu & Toguri (this volume) showed that when (a) $\gamma_{\rm drop} > \gamma_{\rm pool}$ the droplet penetrated into the pool and (b) $\gamma_{\rm drop} < \gamma_{\rm pool}$ the drop will spread out over the surface.

9. Conclusions

(i) Variable weld penetration in TIG/GTA, laser and electron beam welding is caused by the direction and the magnitude of the fluid flow in the weld pool.

(ii) For normal welding conditions the fluid flow in the weld pool is controlled by the thermocapillary forces.

(iii) In most steels the direction of thermocapillary flow is determined by the sulphur content of the steel.

(iv) Other associated welding problems such as 'arc wander' and 'porosity' are also affected by the direction of fluid flow in the weld pool.

(v) The surface properties of the alloy may also be responsible for 'humping' and 'undercutting' and for the surface rippling of the weld.

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